Dr. Claudia Ratti's Nuclear Theory Group

UH NSM

Dualities probing QCD

Prachi Garella
Advisor: Claudia Ratti

What is a Duality?

Two things that look the same but are secretly different.
From QM we know there is a duality for the description of electrons.

Electrons

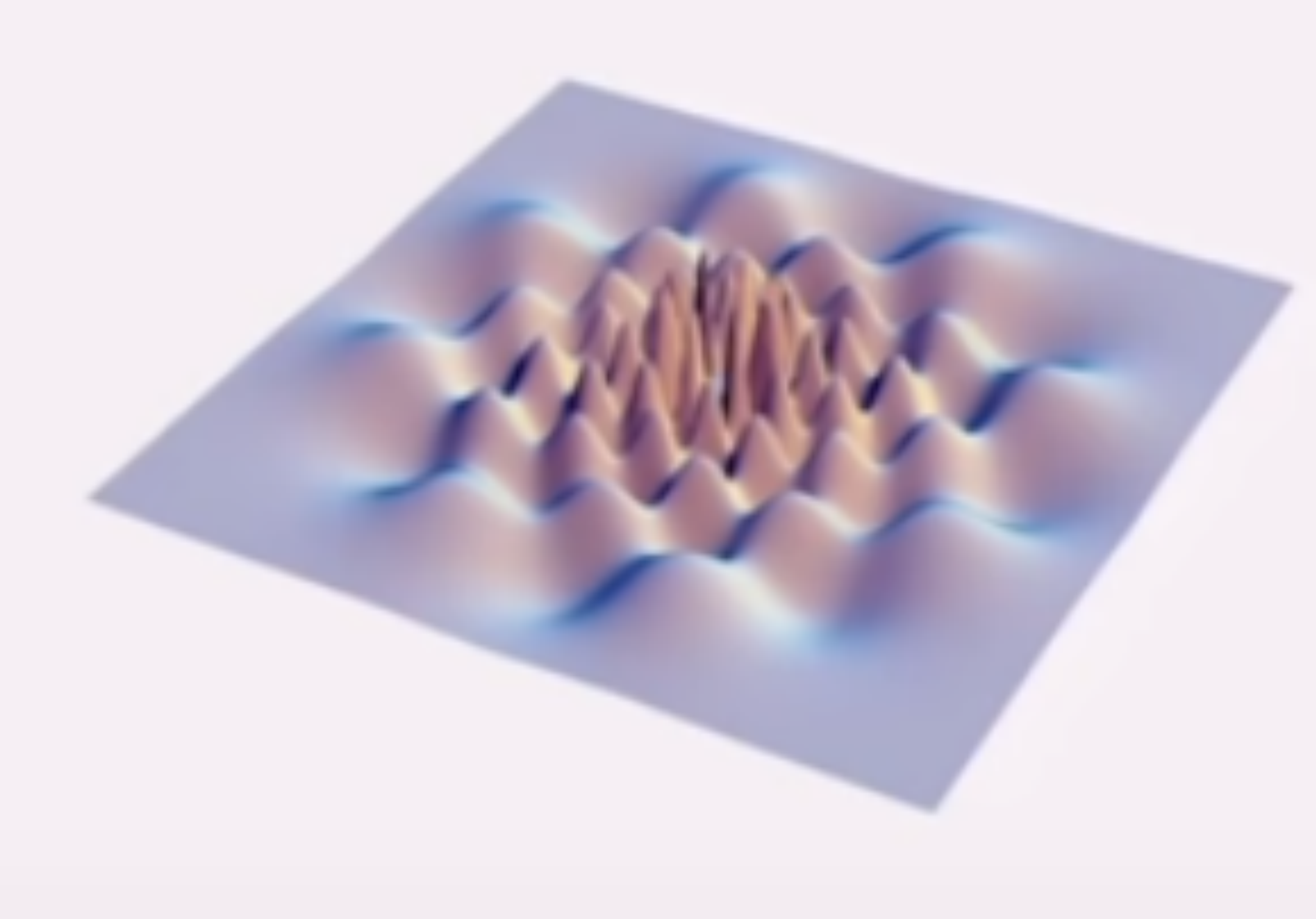
Particles Waves

What is a Duality?

Quantum Gravity

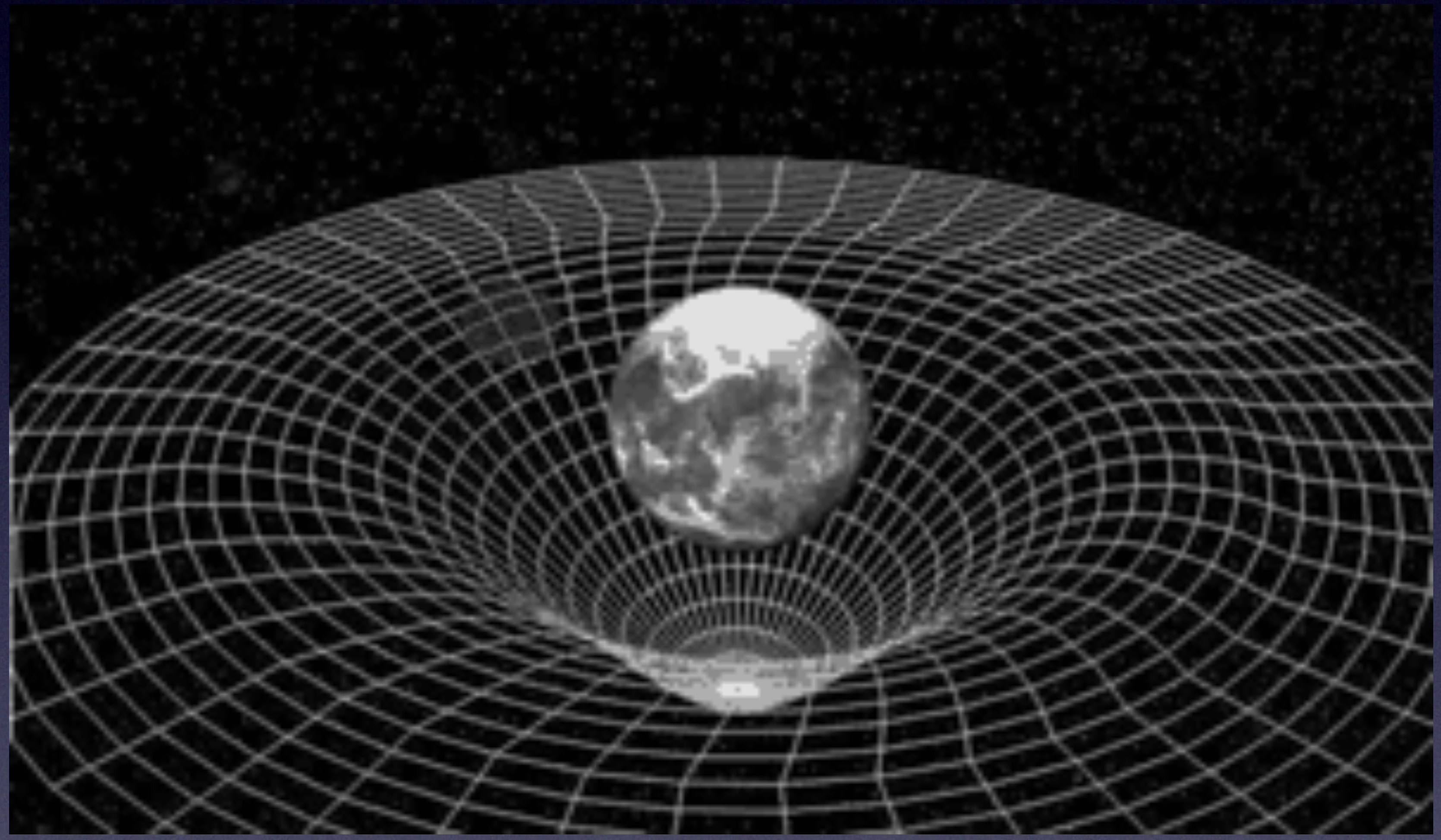
Small Things   Heavy Things

What is a Duality?



QM:
Things fluctuates

+



Space time curves when there is a heavy object

$\Longrightarrow$

Space and time is a dynamical dof that changes when gravity is important

=

?

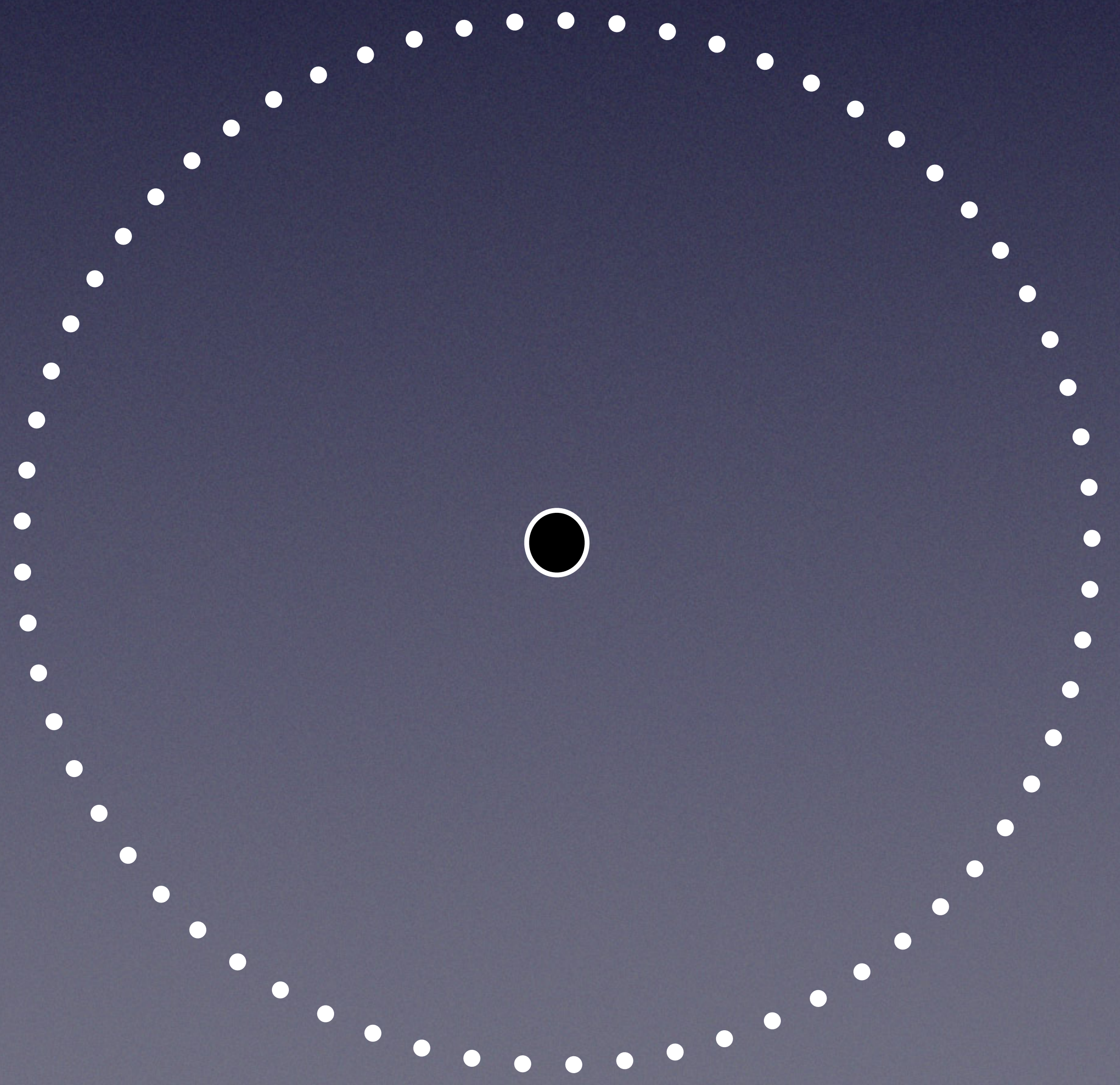
What is a Duality?

Spacetime fluctuates = ?

What is a Duality?

Consider a simple object in GR: black holes (Schwarschild Black hole)

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$



Event Horizon

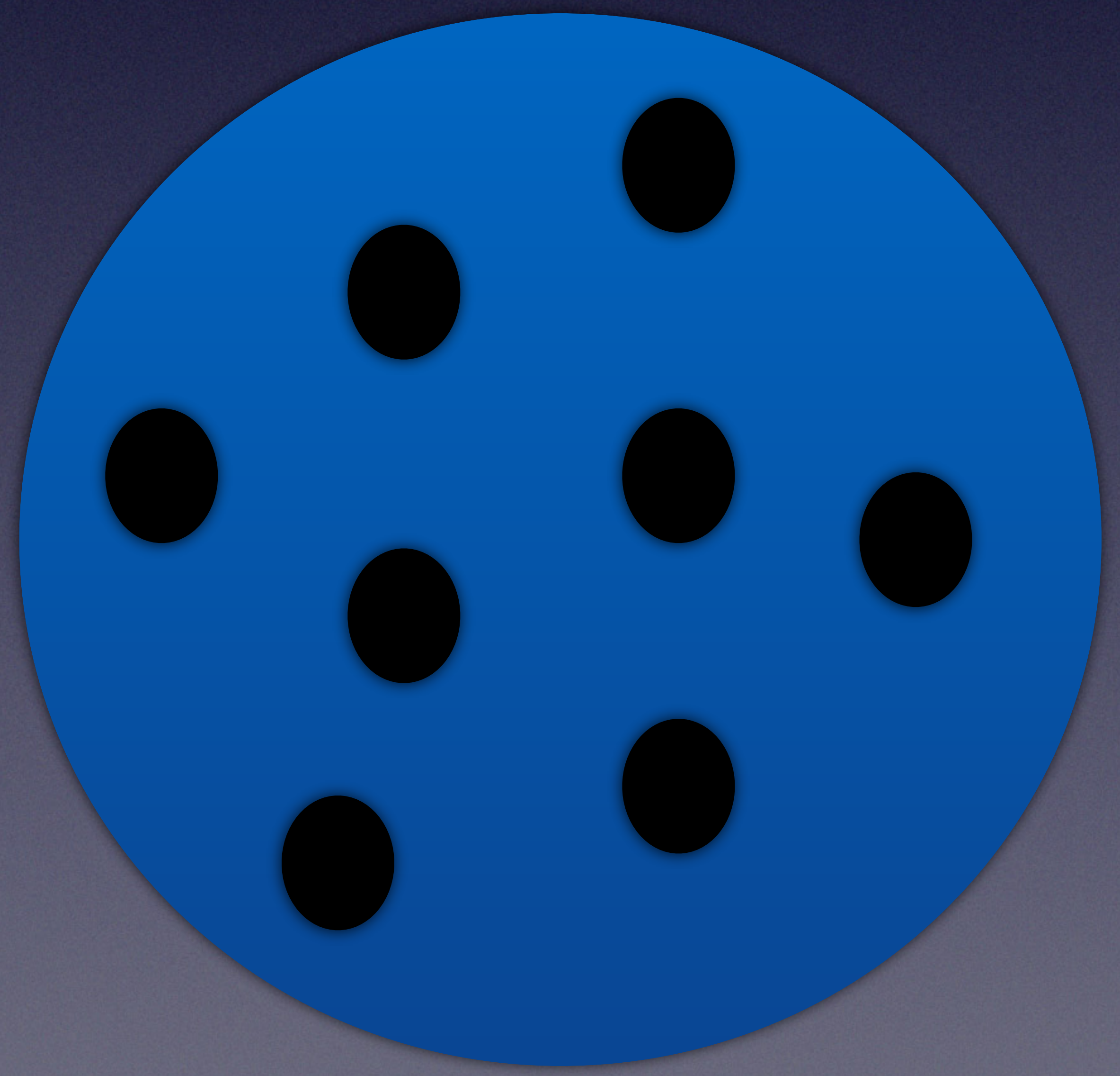
$$r_h = 2GM$$

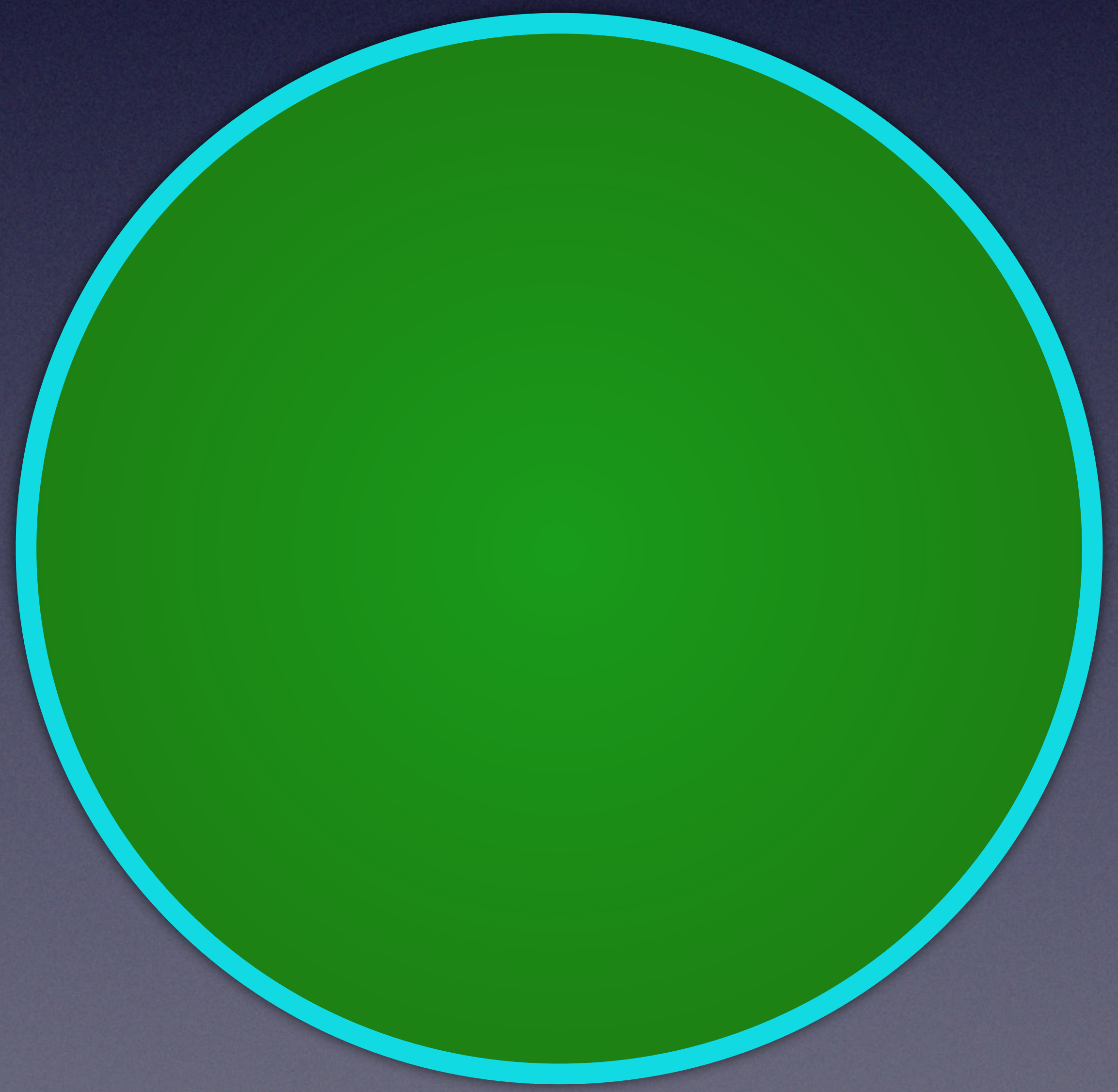
Nothing can escape
from the event horizon

What is a Duality?

Theorem: *A black hole has no hair*

One can completely characterise a blackhole by M, Q, and J

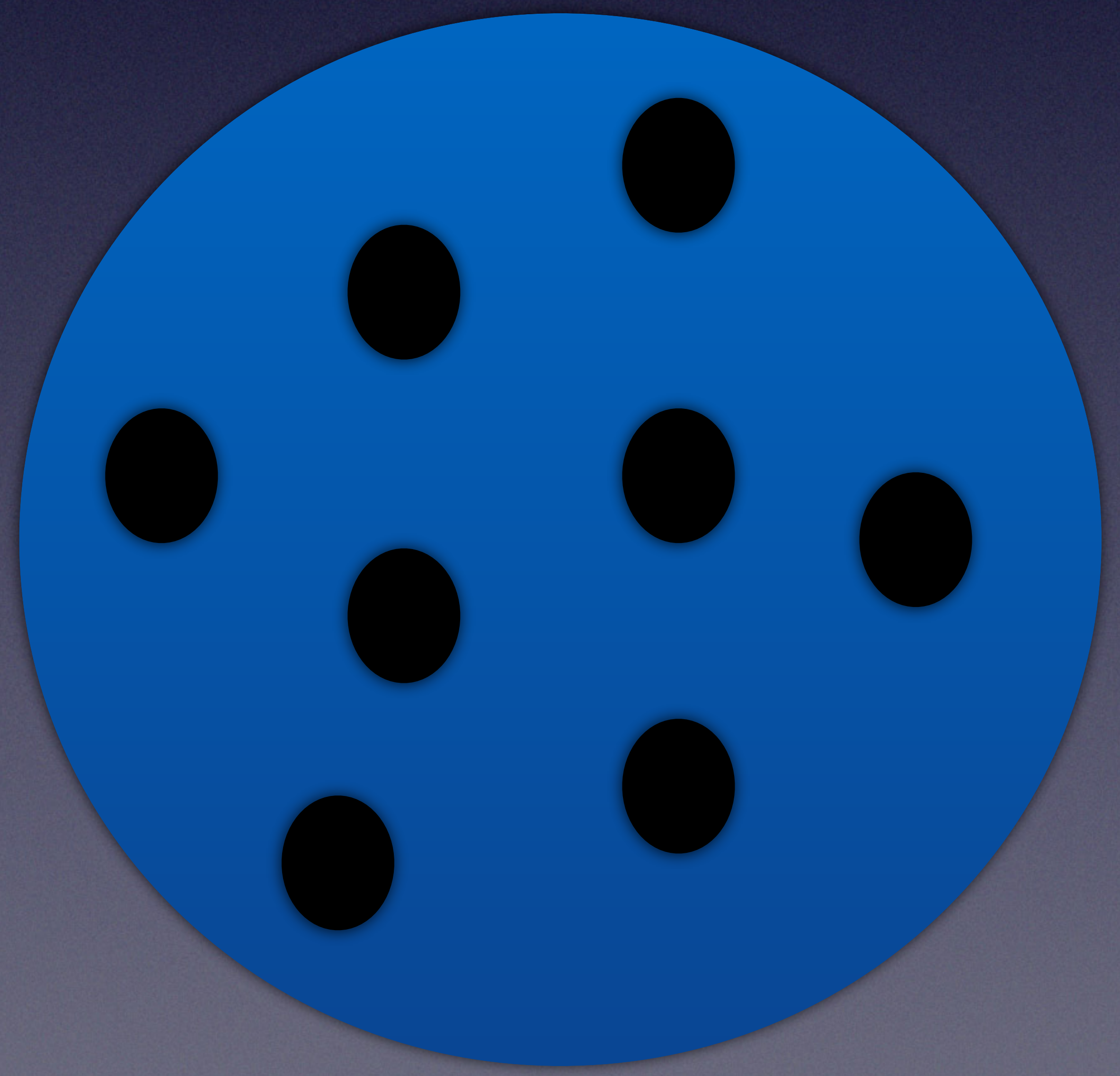




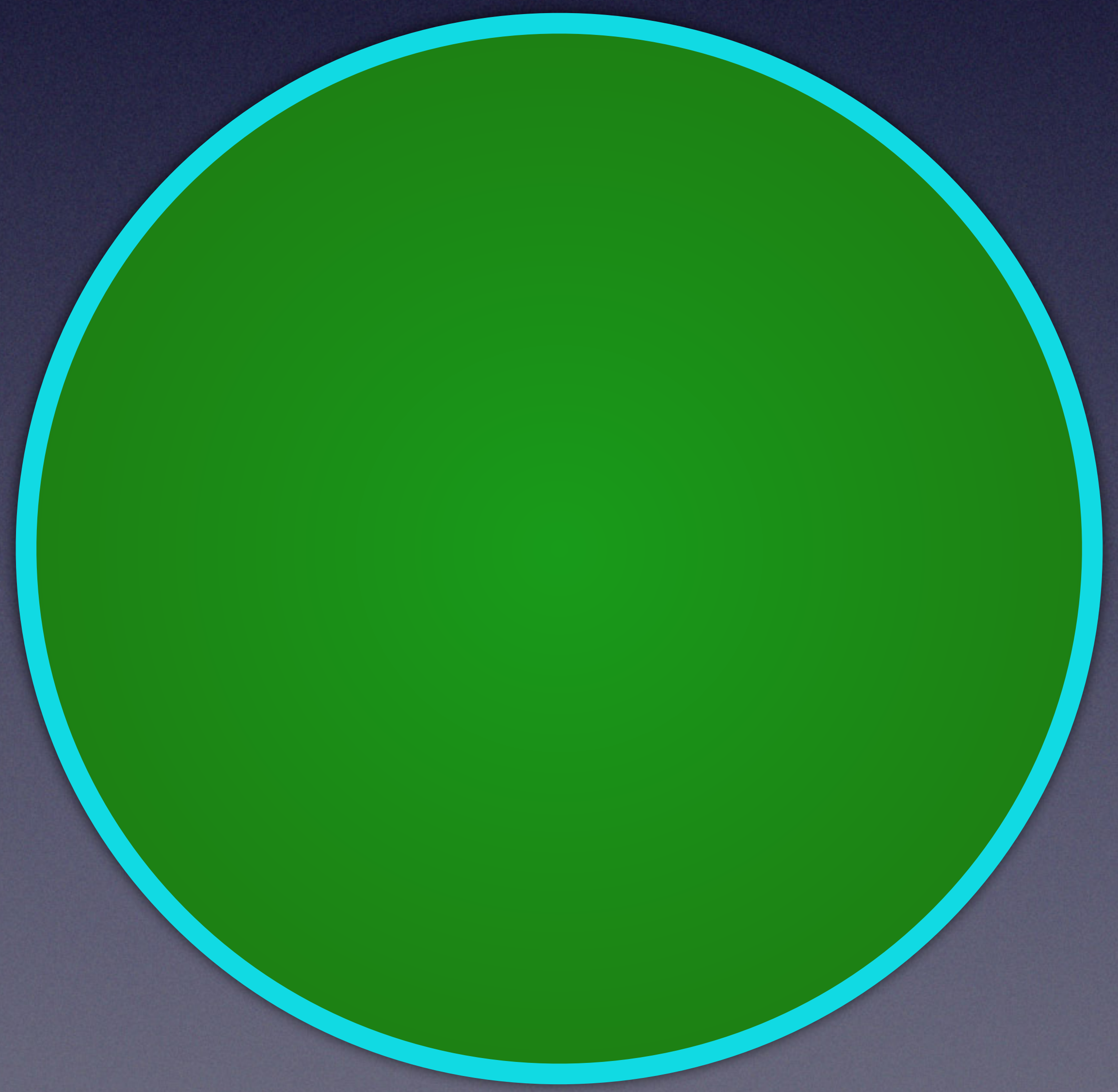
What is a Duality?

Theorem: *A black hole has no hair*

One can completely characterise a blackhole by M, Q, and J



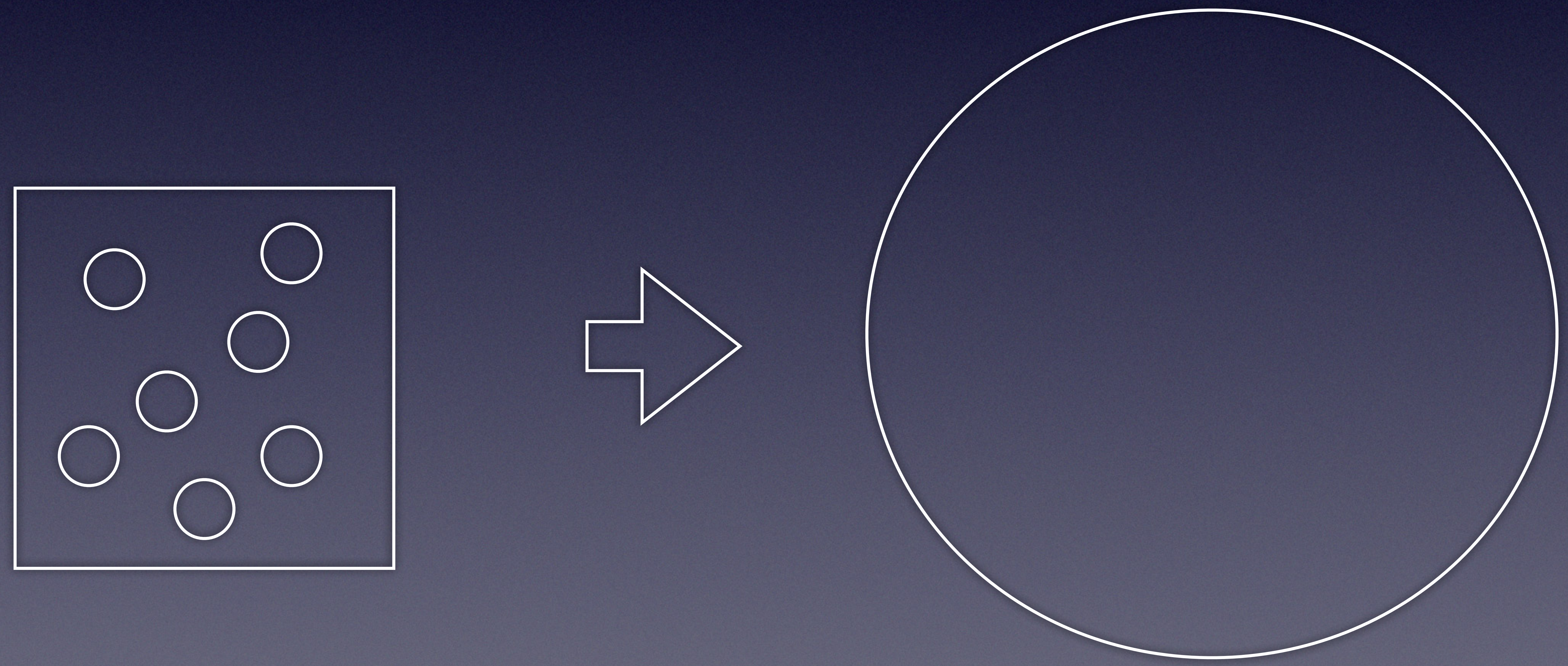
Information about the initial state is lost



What is a Duality?

Take a box of entropy, $S(B) = k_b\, ln\Omega$

Second law: $\Delta S \geq 0$



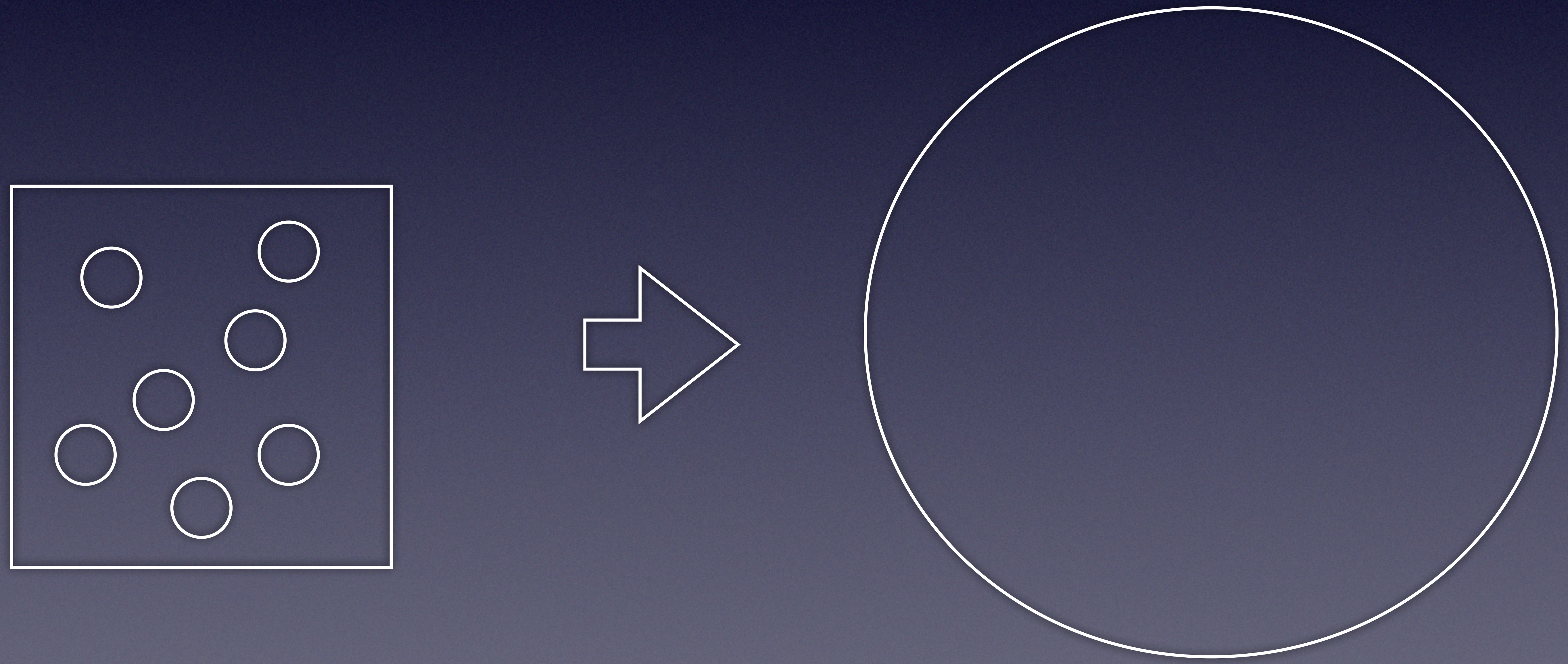
$S(B) \neq 0$

$S_{BH} = k_b\, ln1 = 0$

What is a Duality?

Take a box of entropy, $S(B) = k_b \, ln\Omega$

Second law: $\Delta S \geq 0$



$S(B) \neq 0$

$S_{BH} = k_b \, ln1 = 0$

$\Delta S < 0$

What is a Duality?

Consider the area of a blackhole

$$A = 4\pi(2GM)^2 \sim r_h^2$$

$$d\,M = \frac{1}{8\pi GM}\, d\left(\frac{A}{4G}\right)$$

For any Black hole we have, the first law of black hole mechanics

$$dM = \frac{K}{2\pi}\, d\left(\frac{A}{4G}\right) + \Omega_H dJ + \Phi dQ$$

What is a Duality?

If we take two blackholes and combine their areas,

$$A_1 + A_2 = 4\pi \left[(2GM_1)^2 + (2GM_2)^2\right]$$

$$A_{total} = 4\pi \left[2G(M_1 + M_2)^2\right]$$

$$\Rightarrow \quad A_{total} > A_1 + A_2$$

For any Black hole we have, the second law of black hole mechanics

$$\frac{dA}{dt} > 0$$

What is a Duality?

Black holes

$$dM = \frac{K}{2\pi} d\left(\frac{A}{4G}\right) + \Omega_H dJ + \Phi dQ$$

$$\frac{dA}{dt} > 0$$

Thermodynamics

$$dE = TdS + \mu dN$$

$$\frac{dS}{dt} > 0$$

What is a Duality?

Black holes

Thermodynamics

$$dM = \frac{K}{2\pi} d\left(\frac{A}{4G}\right) + \Omega_H dJ + \Phi dQ$$

$$\frac{dA}{dt} > 0$$

$$E \sim M$$

$$S \sim A$$

$$T \sim K$$

$$dE = TdS + \mu dN$$

$$\frac{dS}{dt} > 0$$

What is a Duality?

Hawking Temperature

$$T_H = \frac{K\hbar}{2\pi}$$

Bekenstein-Hawking Entropy

$$S_{BH} = \frac{A}{4G}\frac{1}{\hbar}$$

What is a Duality?

Counterintuitive

$$S_{BH} \sim A$$

Certain Theories of Quantum Gravity in D dimensions (curved) is exactly equivalent to theories of particles in D-1 dimensions

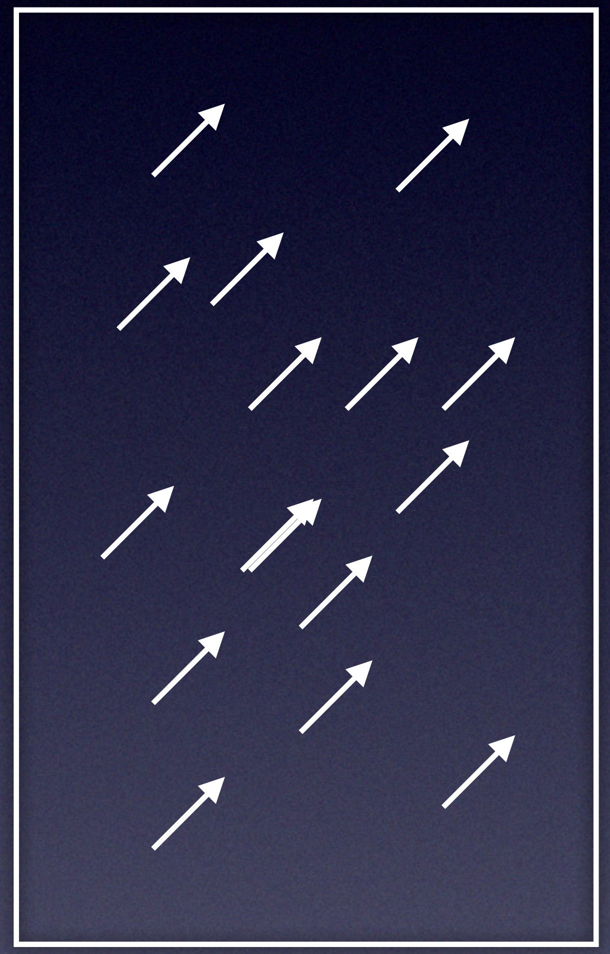
⇨ Holographic Correspondence

AdS/CFT

String Theory on 5d AdS = $\underbrace{\text{SU(N) SYM in 4d}}_{\text{CFT}}$

Degrees of freedom of SU(N) assembles itself into gravity in a higher dimension

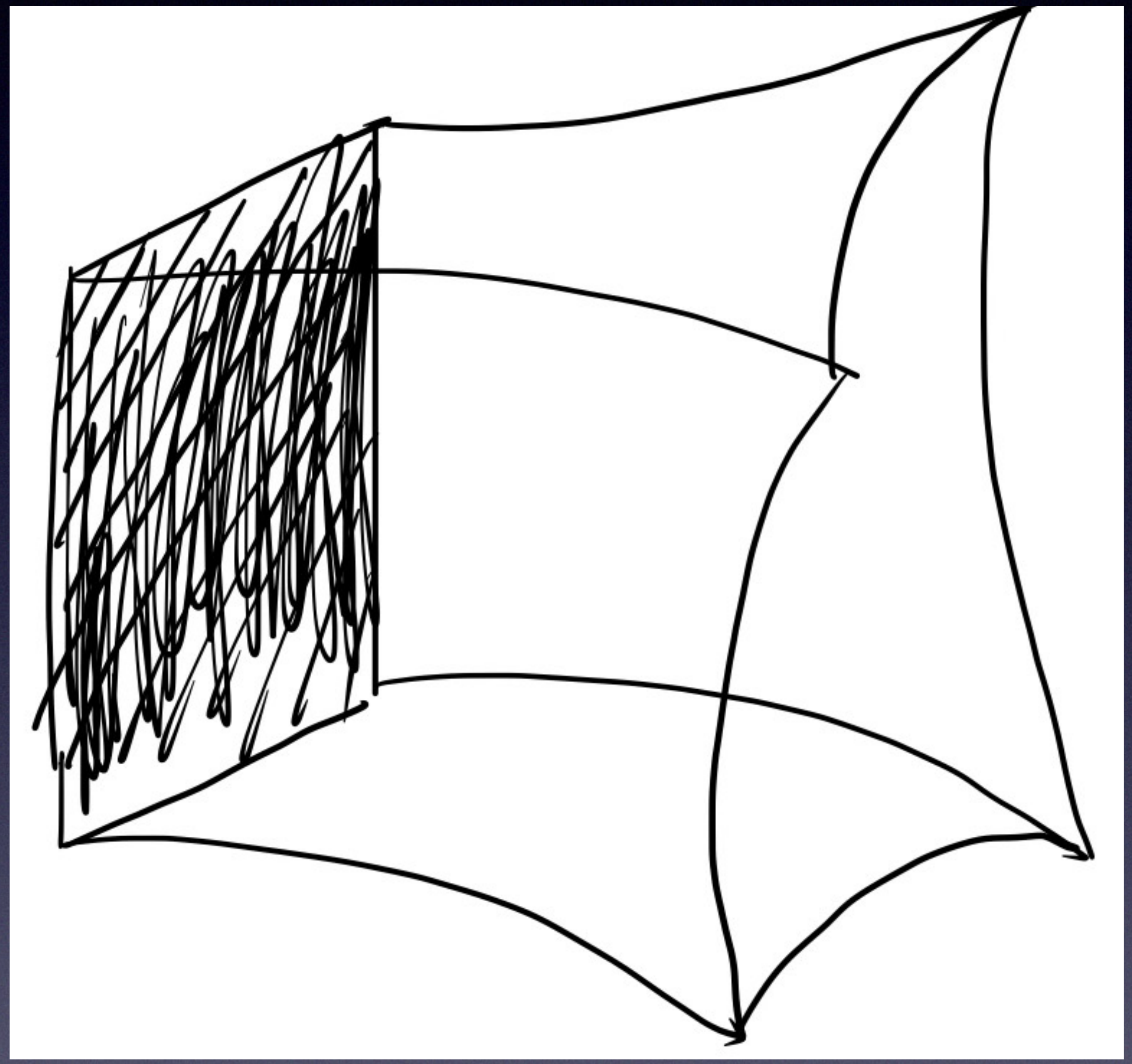
AdS/CFT



Take your particle theory (SU(N)) and heat it up

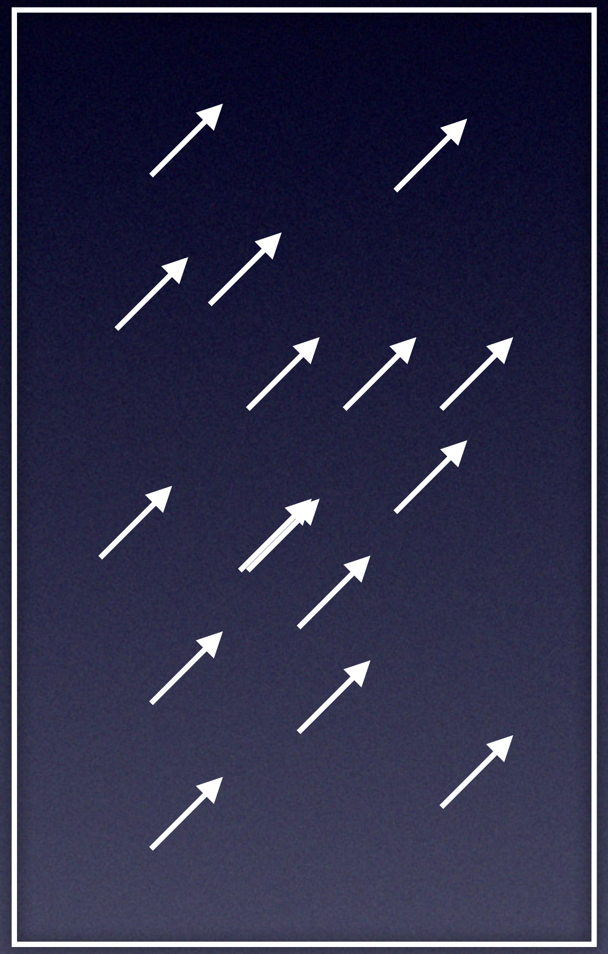
$T \sim K$

$=$



Black hole in the gravitational theory

AdS/CFT

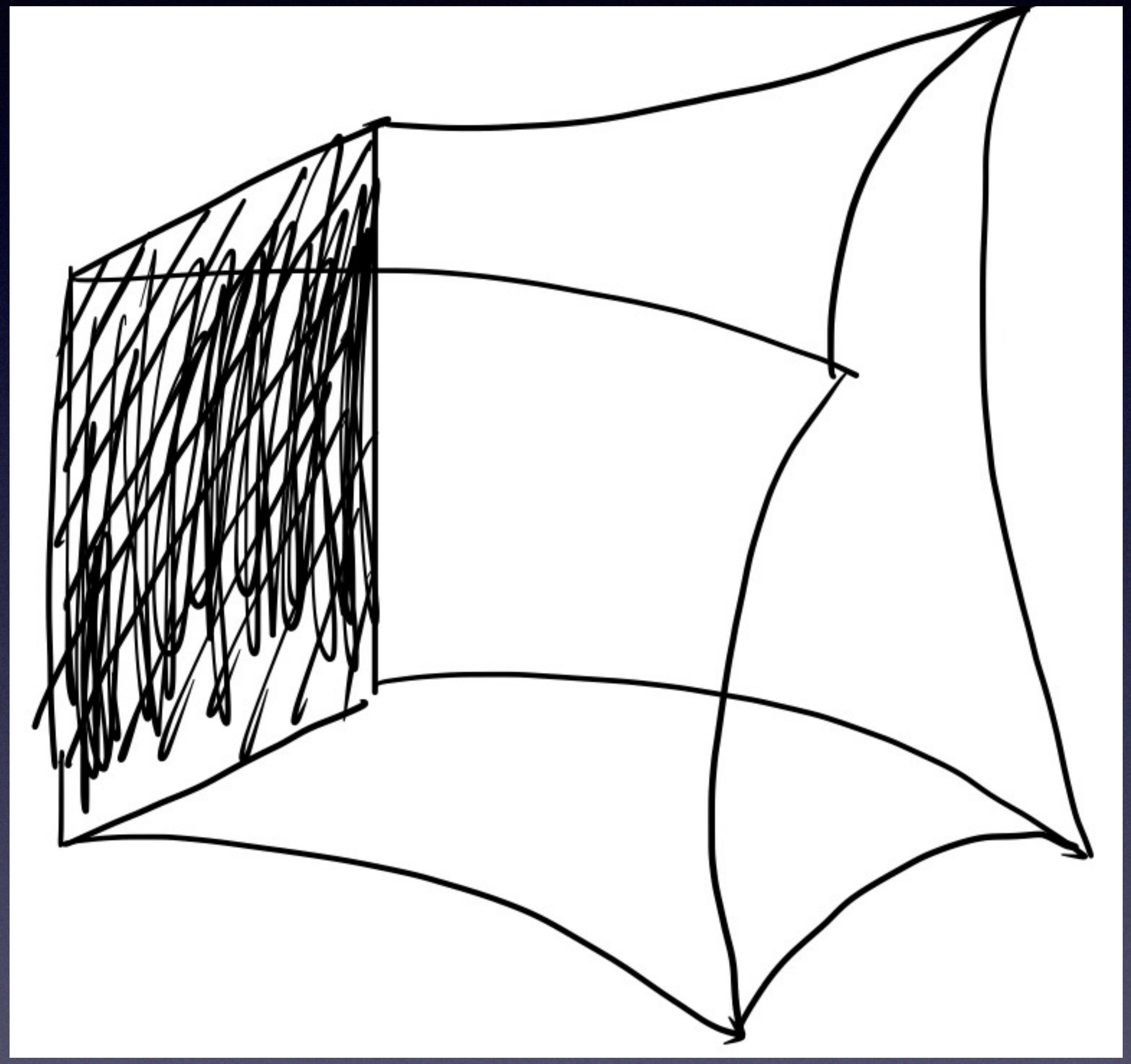


Take your particle theory (SU(N)) and heat it up

$T \sim K$

$=$

Hawking Temperature of the blackhole is the same temperature we put our particle theory at.



Black hole in the gravitational theory

Conformal Field Theory

Any quantum field theory which is scale invariant (rescales length but preserves angle). A QFT which is invariant under the conformal group O(2,D).

Conformal Group		Generators
Translations	$\delta x_\mu = a_\mu$	P_μ
Lorentz Transformations	$\delta x_\mu = \omega_{\mu\nu} x_\nu$	$J_{\mu\nu}$ $(\omega_{\mu\nu} = -\omega_{\nu\mu})$
Dilatation	λx_μ	D
Special Conformal Transformations	$b_\mu x^2 - 2x_\mu (bx)$	K_μ

Conformal Field Theory

Note: Even if a theory is conformally invariant classically, the conformal invariance breaks when we add the quantum corrections (renormalisation scale).

Example: In pure YMT (gauge fields without matter fields) the classical theory is indeed gauge invariant. However when you add the quantum corrections, the gauge coupling g becomes energy scale dependant which is described by the $\beta(g)$. which describes how g changes with respect to energy scale μ.

$$\mu \frac{dg}{d\mu} = \beta(g)$$

Which gives us the running coupling $g(\mu)$

Conformal Field Theory

Because of this we introduce a dimension full parameter Λ_{QCD} which characterises the scale at which the running coupling becomes strong. $T_{\mu\nu} \sim \beta(g) F^2_{\mu\nu}$

The classical dimension d of a field is corrected by an anomalous dimension Δ

$$\Delta = d + r(g)$$

Where $r(g)$ defines
RG flow

This is called dimensional transmutation

AdS Space

Why AdS

- AdS_5 is the maximally symmetric solution of the Einstein equations in 5d with negative cosmological constant.
- AdS_5 shares the $O(2,4)$ symmetry group with CFT_4

AdS Space

Action for gravity in 5 dimensions

$$S = \frac{1}{16\pi G_5} \int dx^5 \sqrt{|g|} (R - \Lambda)$$

In d dimensional spacetime Einstein's field equations are,

$$R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R = -\frac{\Lambda}{2} g_{\mu\nu}$$

Here, we can substitute $R = \frac{5}{3}\Lambda$,

$$R_{\mu\nu} = \frac{\Lambda}{3} g_{\mu\nu}$$

AdS/CFT correspondence (Formulation)

- Both CFT_4 and AdS_5 have the $O(2,4)$ symmetry.
- In QG number of degrees of freedom of a region of space time grows with the area of the region and not the volume.
- (As a consequence) All dynamics in AdS_5 can be reformulated as a boundary effect and it is captured by a 4D local field theory.

AdS/CFT correspondence (Formulation)

What do we need?

- We need a map between observables in the two theories
- Prescription for comparing physical quantities and amplitudes

Effective Action of the bulk

$$S_{AdS_5} = (g_{\mu\nu}, A_\mu, \phi, \dots)$$

*We assume a potential for the scalar field with a negative value at the minimum thus creating a negative cosmological constant for AdS_5 vacuum.

Lagrangian of the boundary is $\mathscr{L}_{CFT}$

AdS/CFT correspondence (Formulation)

<u>Correspondence:</u> A field h in AdS_5 is associated with an operator in CFT with the same quantum numbers and know about each other via boundary couplings.

Every operator $\hat{O}$ in CFT_4 can be associated to an operator h,

$$\mathscr{L}_{CFT} + \int d^4x \, h\hat{O}$$

Let's say you have a 5 dimensional field $h(x, x_5)$ where $h(x)$ is the boundary value.

For every source configuration $h(x)$ there is a 5 dimensional field configuration $h(x, x_5)$ such that it solves the 5 dimensional equation of motion derived by S_{AdS_5}

AdS/CFT correspondence (Formulation)

Fundamental statement of AdS/CFT correspondence

$$e^{W(h)} = \left\langle e^{\int hO} \right\rangle_{QFT} = e^{S_{AdS_5}}$$

Where W (h) is defined as the functional generator for connected correlation functions of O

AdS/CFT correspondence (Formulation)

Field h that couples to an operator $\hat{O}$ have the same quantum numbers under isometry $O(2,4)$, thus we can find the coupling between bulk fields and boundary operators using symmetries.

Example: If we introduce a gauge field by covariantizing the boundary action we get,

$$\mathscr{L}_{CFT} + \int d^4x \sqrt{g} (g_{\mu\nu} T_{\mu\nu} + A_\mu J_\mu + \phi F_{\mu\nu}^2 + \dots)$$

AdS/CFT correspondence (Formulation)

$$\mathscr{L}_{CFT} + \int d^4x\sqrt{g}(g_{\mu\nu}T_{\mu\nu} + A_\mu J_\mu + \phi F^2_{\mu\nu} + \dots)$$

Operator		Source
CFT (Boundary)	⟺	AdS (Bulk)
$T_{\mu\nu}$ Stress energy tensor		Graviton
J_μ Current		A_μ gauge fields
Scalar operator $O = Tr(F^2_{\mu\nu})$		Dilaton ϕ

The conservation of the stress energy tensor or currents in the CFT corresponds to gauge invariance in AdS.

AdS/QCD

QCD: Strongly coupled Quantum field theory of partons, which are quarks (fermions) and gluons (non-abelian gauge fields)

How do we create a duality between AdS and QCD?

There are two approaches for any holographic mode

1. Top-down: We know the bulk theory and thus determine the boundary theory.
2. Bottom-up: We construct the bulk theory in order to get the closest real-physical system as the dual theory.

Holographic Model for Hot and Baryon Dense QGP

- To obtain AdS/QCD from AdS/CFT we need to break conformal invariance.
- To obtain a nonconformal system is where the free parameters of the model are constrained by existing results from Lattice QCD in some specific regime.
- Once the parameters are fixed, one can then use this model to make predictions.

We aim to construct an approximate holographic dual for the equation of state of QGP without trying to implement confinement, chiral symmetry breaking at low temperatures, and asymptotic freedom at asymptotically high temperatures.

Holographic Model for Hot and Baryon Dense QGP

We aim to construct an approximate holographic dual for the equation of state of QGP without trying to implement confinement, chiral symmetry breaking at low temperatures, and asymptotic freedom at asymptotically high temperatures.

Holographic Model for Hot and Baryon Dense QGP

The QCD equation of state is used to fix the free parameters at finite temperature and vanishing chemical potentials. By fixing the free parameters using these specific LQCD data the resulting thermodynamic quantities and transport coefficients can be predicted.

Bulk metric + Maxwell field + Dilaton Field

Bulk metric	Maxwell field	Dilaton Field
Holographically dual to boundary QFT energy momentum tensor	Boundary value of time component gives chemical potential at dual QFT	A real scalar field to break conformal symmetry

Holographic EOS

$$S_{bulk} = \frac{1}{2\kappa_5^2} \int_{\mathcal{M}_5} d^5x \sqrt{-g} \left[R - \frac{(\partial\phi)^2}{2} - V(\phi) - \frac{f(\phi) F_{\mu\nu}^2}{4} \right]$$

1. Einstein-Hilbert term with a negative cosmological constant for the metric $g_{\mu\nu}$.
2. Kinetic terms for the abelian gauge fields A_μ
3. Dilaton field ϕ
4. Arbitrary potential $V(\phi)$ for the dilaton
5. Interaction term between Maxwell and dilaton fields, which as a function $f(\phi)$

Holographic EOS

$V(\phi)$, $f(\phi)$, G_5, and the characteristic energy scale $\Lambda \propto L^{-1}$ need to be dynamically fixed by holographically matching the specific set of LQCD results.

Holographic EOS

Equation of motions

$$\phi''(r) + \left[\frac{h'(r)}{h(r)} + 4A'(r)\right]\phi'(r) - \frac{1}{h(r)}\left[\frac{\partial V(\phi)}{\partial\phi} - \frac{e^{-2A(r)}\Phi'(r)^2}{2}\frac{\partial f(\phi)}{\partial\phi}\right] = 0,$$

$$\Phi''(r) + \left[2A'(r) + \frac{d[\ln f(\phi)]}{d\phi}\phi'(r)\right]\Phi'(r) = 0,$$

$$A''(r) + \frac{\phi'(r)^2}{6} = 0,$$

$$h''(r) + 4A'(r)h'(r) - e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

$$h(r)[24A'(r)^2 - \phi'(r)^2] + 6A'(r)h'(r) + 2V(\phi) + e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0,$$

Holographic EOS

We have two sets of coordinates, standard coordinates and numerical coordinates. Numerical coordinates corresponds to rescaling of standard coordinates to specify definite numerical values for the radial location of the black hole horizon.

Holographic EOS

- We then solve the bulk equations of motion for different pairs of initial conditions ϕ_0, Φ_1 (dilaton field, radial derivative of the Maxwell field).
- Which helps us to find numerical solutions for the EMD fields in thermal equilibrium that are associated with definite thermal states at the boundary QFT, through the holographic dictionary.
- The temperature T, the baryon chemical potential μ_B, the entropy density s, and the baryon charge density ρ_B of the medium are given by,

Holographic EOS

$$T = \frac{\sqrt{-g'_{\tilde{t}\tilde{t}} g^{\tilde{r}\tilde{r}\,\prime}}}{4\pi}\bigg|_{\tilde{r}=\tilde{r}_H} \Lambda = \frac{e^{\tilde{A}(\tilde{r}_H)}}{4\pi} |\tilde{h}'(\tilde{r}_H)| \Lambda = \frac{1}{4\pi \phi_A^{1/\nu} \sqrt{h_0^{\rm far}}} \Lambda,$$

$$\mu_B = \lim_{\tilde{r}\to\infty} \tilde{\Phi}(\tilde{r}) \Lambda = \frac{\Phi_0^{\rm far}}{\phi_A^{1/\nu} \sqrt{h_0^{\rm far}}} \Lambda,$$

$$s = \frac{S}{V} \Lambda^3 = \frac{A_H}{4 G_5 V} \Lambda^3 = \frac{2\pi}{\kappa_5^2} e^{3\tilde{A}(\tilde{r}_H)} \Lambda^3 = \frac{2\pi}{\kappa_5^2 \phi_A^{3/\nu}} \Lambda^3,$$

$$\rho_B = \lim_{\tilde{r}\to\infty} \frac{\partial \mathcal{L}}{\partial(\partial_{\tilde{r}} \tilde{\Phi})} \Lambda^3 = -\frac{\Phi_2^{\rm far}}{\kappa_5^2 \phi_A^{3/\nu} \sqrt{h_0^{\rm far}}} \Lambda^3,$$

Holographic EOS

The dimensionless reduced second order baryon susceptibility,

$$\hat{\chi}_2^B \equiv \frac{\chi_2^B}{T^2} \equiv \frac{\partial^2(P/T^4)}{\partial(\mu_B/T)^2}$$

When evaluated at $\mu_B = 0$, the integral form is given by,

$$\hat{\chi}_2^B(T, \mu_B = 0) = \frac{1}{16\pi^2} \frac{s}{T^3} \frac{1}{f(0) \int_{r_H}^{\infty} dr \; e^{-2A(r)} f(\phi(r))^{-1}},$$

Holographic EOS

Pressure of the dual QFT fluid is

$$P(T, \mu_B) \approx \int_{T_{\text{low}}}^{T} dT\, s(T, \mu_B),$$

The energy density of the medium can be calculated from the thermodynamic relation,

$$\epsilon(s, \rho_B) = Ts(T, \mu_B) - P(T, \mu_B) + \mu_B \rho_B(T, \mu_B).$$

Holographic EOS

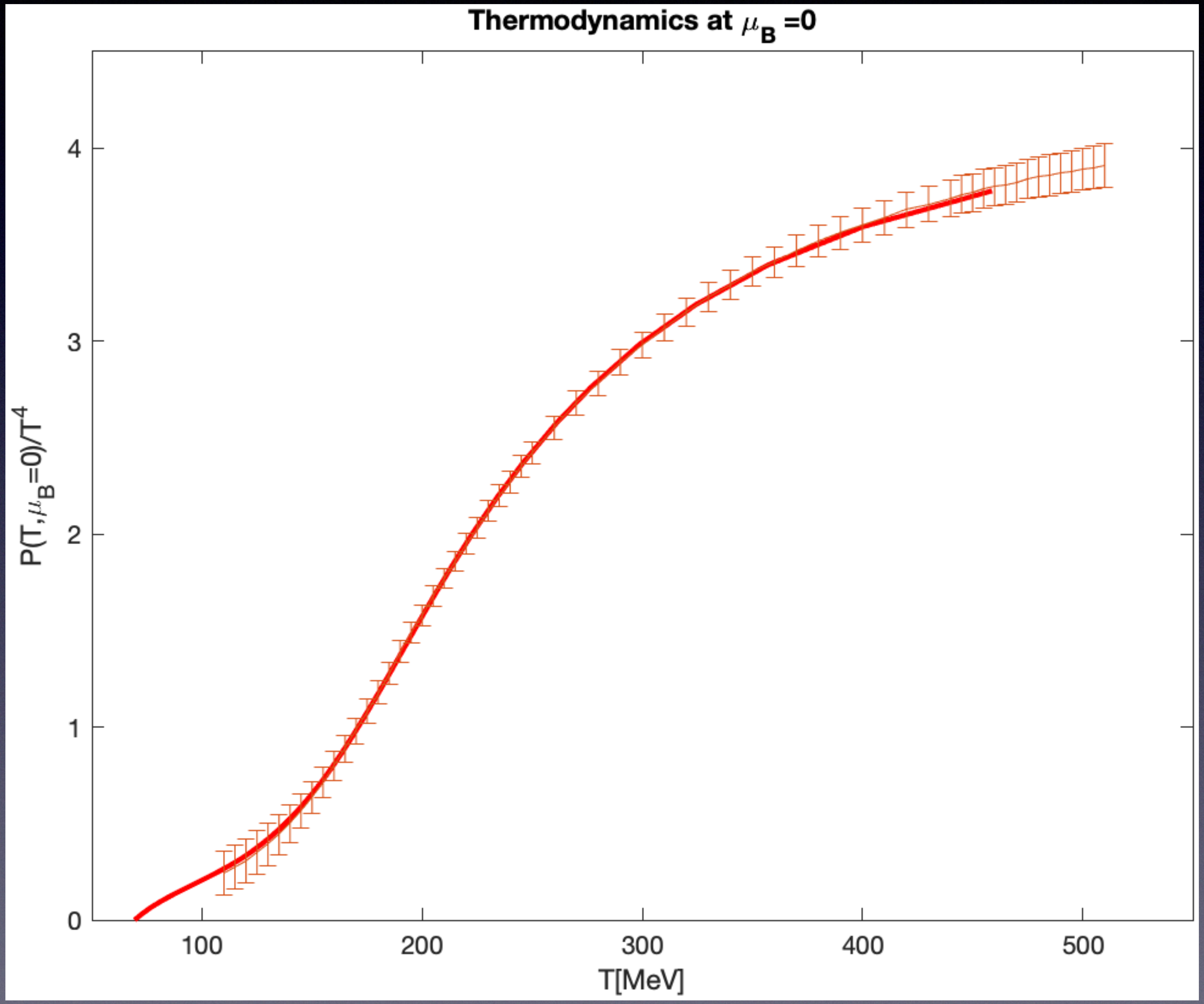
Trace anomaly of the energy momentum tensor also called the interaction measure of the dual QFT at the boundary is given by

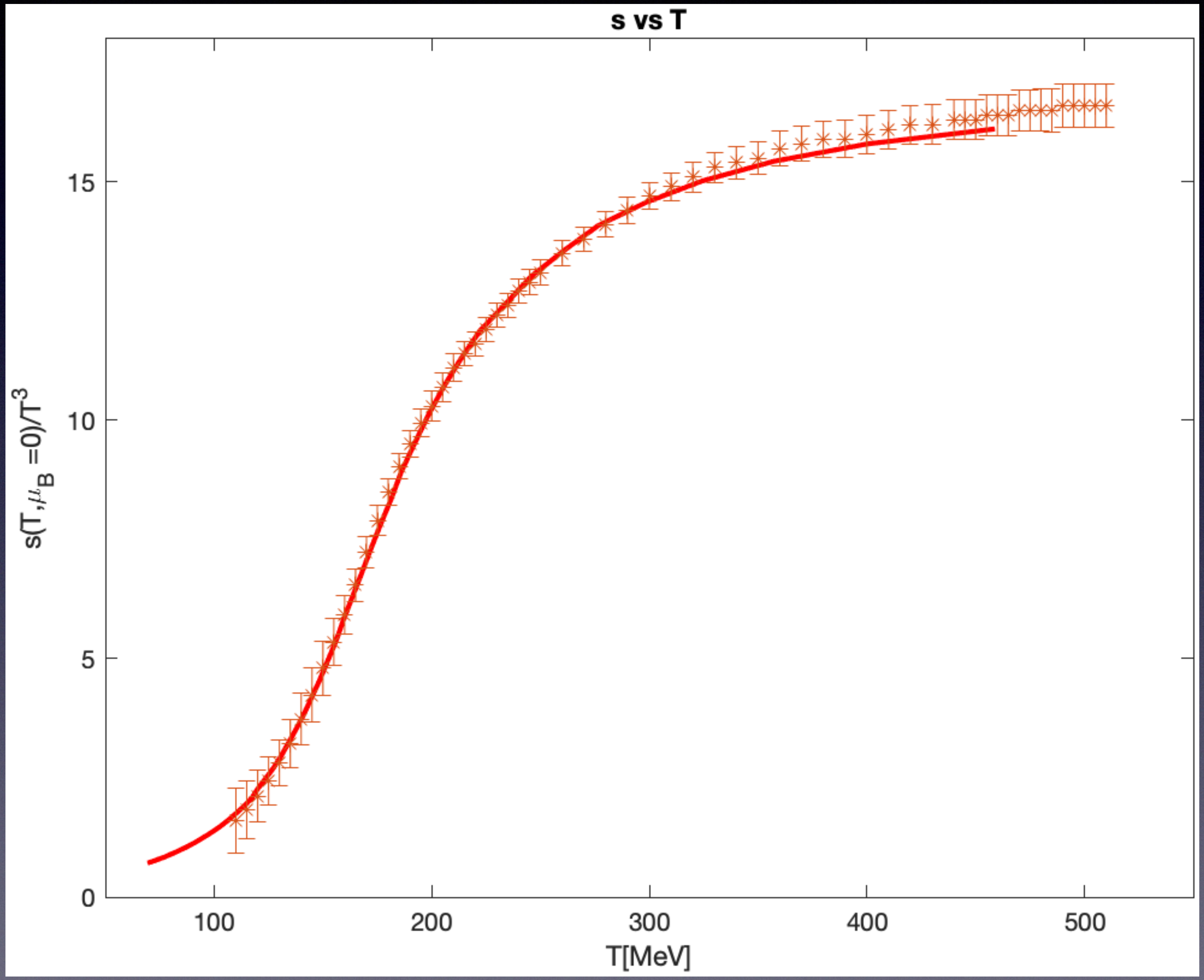
$$I(T,\mu_B) = \epsilon(T,\mu_B) - 3P(T,\mu_B).$$

The speed of sound squared in terms of the derivatives of T, μ_B

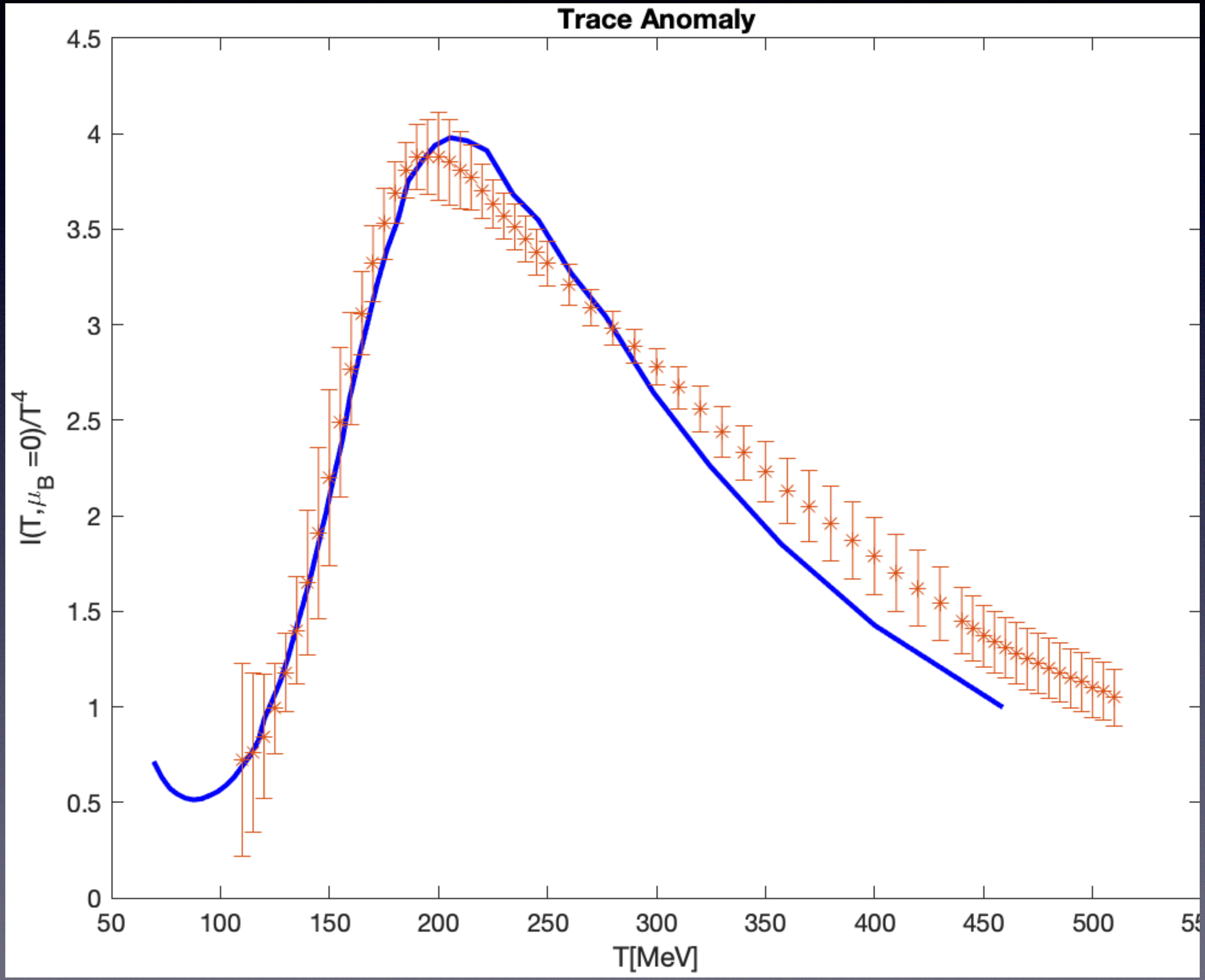
$$\left[c_s^2(T,\mu_B)\right]_{s/\rho_B} = \frac{\rho_B^2 \partial_T^2 P - 2s\rho_B \partial_T \partial_{\mu_B} P + s^2 \partial_{\mu_B}^2 P}{(\epsilon + P)[\partial_T^2 P \partial_{\mu_B}^2 P - (\partial_T \partial_{\mu_B} P)^2]}$$

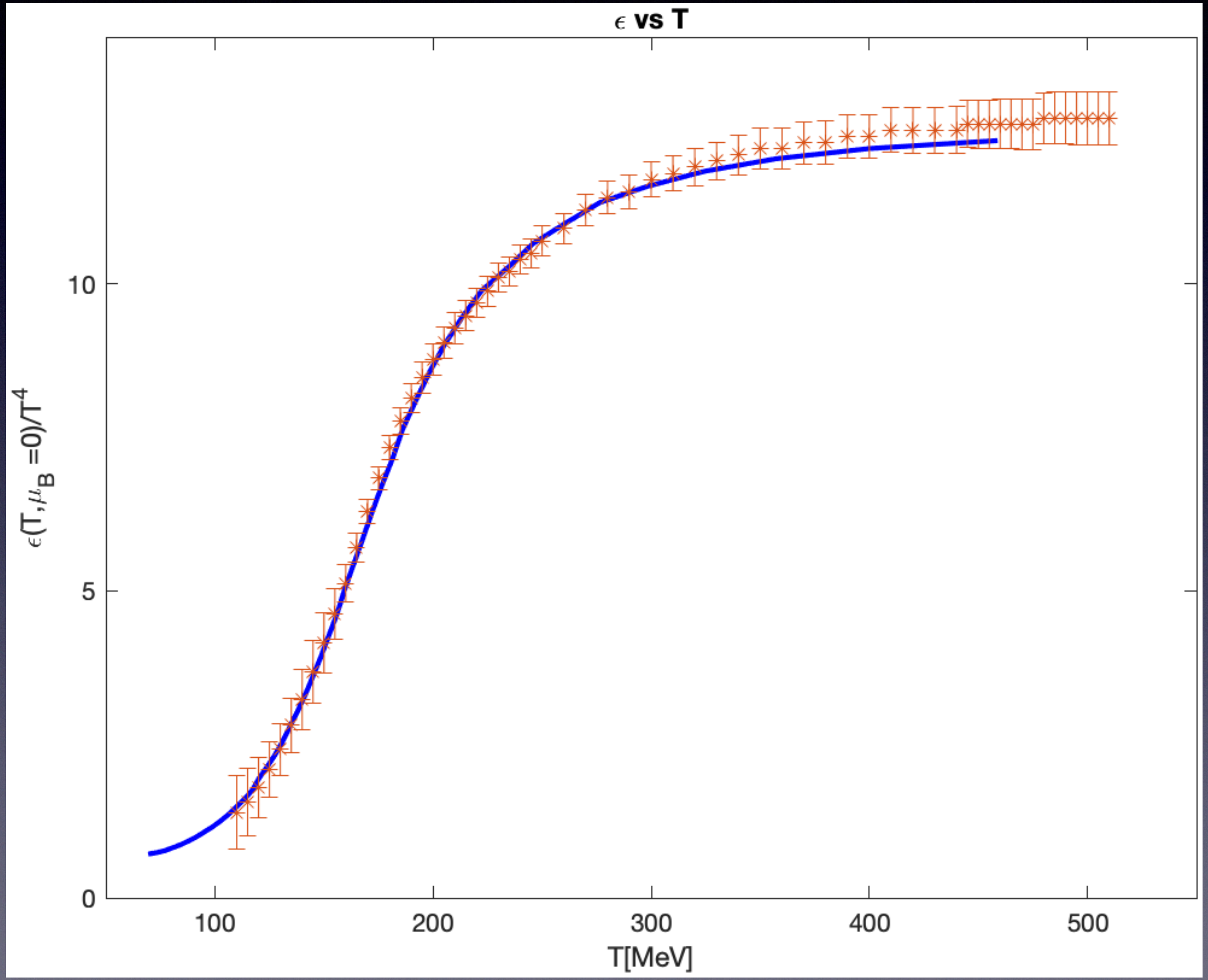
Holographic EOS



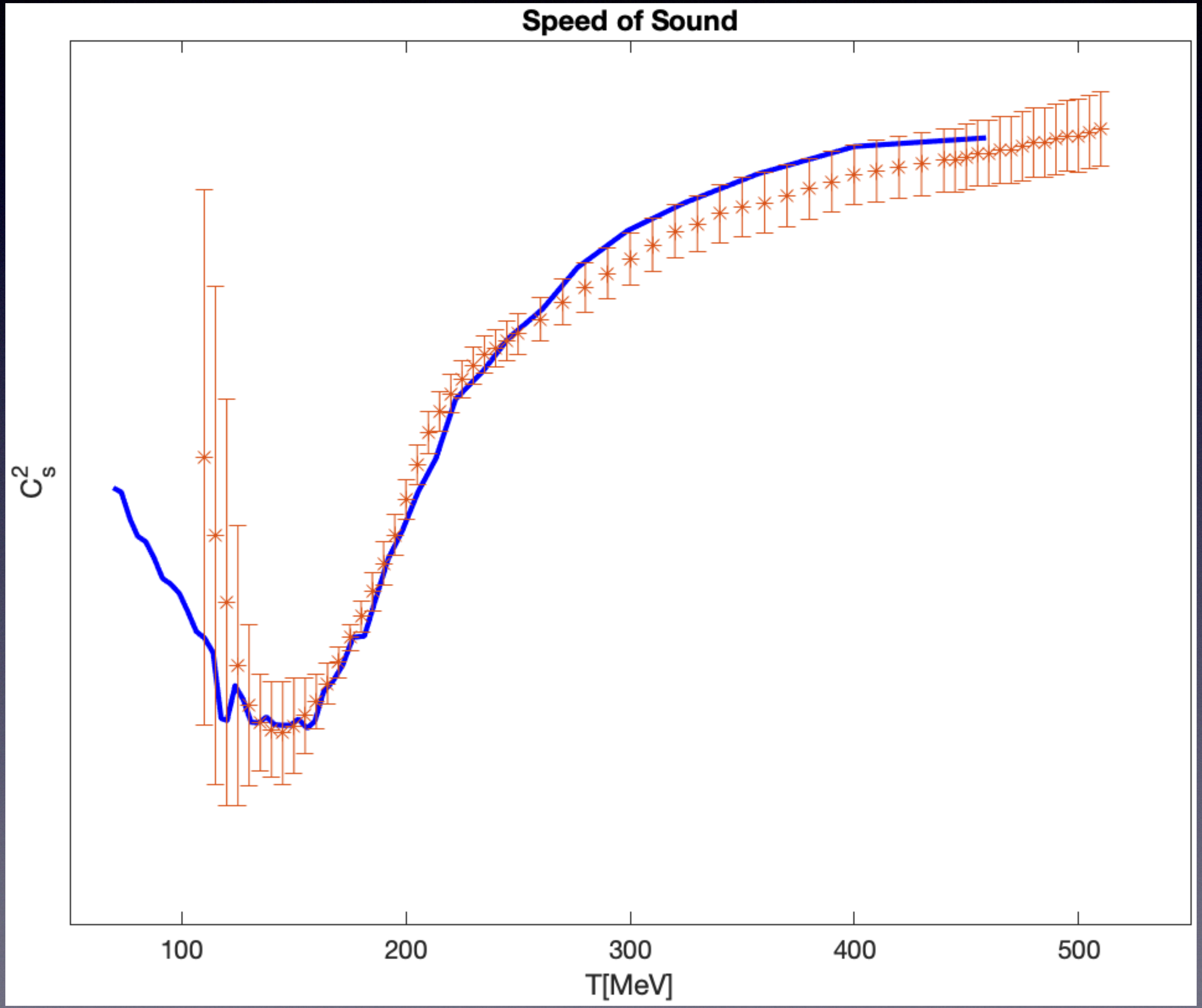


Holographic EOS





Holographic EOS



References

1. J.M. Maldacena, "The large N limit of Superconformal Field Theories and Supergravity"

2. A. Zaffaroni, "Introduction to the AdS/CFT correspondence"

3. R. Rougemont J. Grefa, M. Hippert, J.Noronha, J. Noronha-Hostler I.Portillo, C. Ratti, "Hot QCD Phase Diagram From Holographic Einstein-Maxwell-Dilaton Models"

4. S. Aronson, "The Complexity of Quantum States and Transformations: From Quantum Money to Black Holes"

5. J. D.Bekenstein, "Black holes and the second law"

"Thank you"

Website: www.prachigarella.com

Email: physics@prachigarella.com, pgarella@uh.edu