Blackhole Information

A Paradox

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April 14, 2024

Contents

1 What is a Blackhole?

Blackholes are astronomical objects that have enormous amount of mass packed in a small volume. Such a dense object creates such a strong gravitational pull that not even light can escape from it.

Blackholes have two parts. The first is **Event Horizon** also known as the *point of no return*. An *Event Horizon* is a region of spacetime where gravity becomes so strong that beyond which nothing can escape and will inevitably fall inside (hence, point of no return). Once you fall into the blackhole you will find the second part, **Singularity** of the blackhole. A *singularity* is a point of spacetime that is extremely small and extremely dense. Due to the dense nature of the black hole it distorts spacetime, as a result of which even the fastest thing in the world (light) cannot escape its pull.

Due to the mysterious nature of blackholes, there are several puzzles attached to its mechanics. In this review we will go through three of those puzzles. At the end of the review we will also discuss briefly the current state of our understanding and some recent approaches that claim to solve the paradox. The reader is encouraged to go over the references at the end for further understanding.

2 Black hole Information Paradox

There have been many revolutions in physics but the two main, have been the discovery of *General Relativity* and *Quantum Mechanics*. Therefore, a third great revolution will come along with a complete theory of the quantum description of gravity. Black holes are a great playground to test and examine different quantum gravity theories and therefore physicists are spending a lot of time thinking about the paradoxes that are associated with it.

2.1 Thermodynamics and Reversibility

One of the major criteria to check whether a theory is correct is its consistency with thermodynamics. If a theory is consistent with the laws of thermodynamics we know we are on the right path. However when it comes to black holes, they seem quite inconsistent with the second law of thermodynamics. And if that's true, our established logic says we should discard black holes all together. But black holes are observed objects, how is that possible. Let us see what is going on. The irony of a black hole is that even being so dense, in thermal equilibrium, we can completely characterise a black hole with just three quantities, its mass **M**, charge **Q** and angular momentum **J**. This is called the **No Hair Theorem**. Now the second law of thermodynamics says that the entropy of the universe always increases. Assume Alice drops a box of positive entropy from the universe into the Black hole which causes the entropy of the exterior world to decreases and the entropy of the black hole is given by,

$$S_{bh} = \kappa \ln \Omega.$$

Now according to the *No Hair Theorem*, since a black hole can be characterised completely by its mass, charge and angular momentum $\Omega = 1$[1],the entropy of the black hole turns out to be,

$$S_{bh} = \kappa \ln 1 = 0$$

Jacob Bekenstein tried to rectify this problem by modifying the second law as; 'Common entropy plus black hole entropy never decreases.' [**2**]. Here by common entropy we mean black holes's exterior entropy and now the S_{bh} is given by,

$$S_{bh} = \eta k L_p^{-2} A,$$

where η and k are constants, L_p is the Planck length and A is the area of the event horizon. Bekenstein says that the second law is well defined only by this definition of the Black hole entropy. Now you may ask how does S_{bh} being proportional to the area, solve the problem. Area of a black hole is given by,

$$A = 4\pi(2GM)^2$$

Stephen Hawking claimed that the area of a Black hole's event horizon is an ever increasing quantity for all except for a special class of Black hole transformations[2]. Consider two black holes A and B and let them collide and form a bigger black hole. The area of the combined black hole will always be bigger than

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[1] Black hole is characterised by one macrostate

[2] extreme kerr-mann Black holes

the area of individual black holes.

$$A_1 + A_2 = 4\pi(2GM_1)^2 + 4\pi(2GM_2)^2$$

$$A_{total} = 4\pi(2G(M_1 + M_2))^2$$

Therefore as long as entropy is proportional to an ever increasing quantity (area of the black hole) the second law is not violated[3].

This idea further lead Bekenstein to suggest that there is a upper bound on the entropy that a physical system can store. Anything greater than that limit will result into a black hole. So very quickly black holes went from being objects with zero entropy to ones with maximum entropy.

When we think of entropy, we ought to think about temperature since anything that has entropy, must also have a non-zero temperature. This brought Hawking to the idea, that for black holes to have nonzero temperature, it had to be giving off thermal radiation, but how is that possible since we know that nothing can escape the gravitational pull of black holes. With this thought Hawking predicted that an observer in the vicinity of a black hole will witness thermal radiations coming out of the black hole and this radiation is called **Hawking Radiation**. Near the black hole's event horizon, due to quantum fluctuations, virtual particles come into existence for a short period of time before they are annihilated by each other. Virtual particles are a pair of negative and positive energy excitations and in the short period of their existence, if the particle with negative energy happen to falls into the black hole, the annihilation doesn't occur and instead the particle with positive energy seems to come out of the black hole as *Hawking Radiation.* This way due to the negative energy particle falling into the black hole, the black hole loses its mass. The number of bits (entropy) stored in the black hole scales as $A \sim r^2$ and by comparison the evaporation rate of the black hole scales as $r^3 \sim A^{3/2}$, where r is the radius of the event horizon. Hawking's idea[4] led to a further prediction where he claimed that the radiation coming out of the Black hole is *thermal.*

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[3] A side note to the reader: Naively one might think that the entropy of the such a dense object as a Black hole should scale as the volume, however this counter-intuitive approach of entropy scaling as the area further leads to a more fundamental description of nature, dualities, which gave rise to the much celebrated work of Maldecena, known as the AdS/CFT correspondence, where a d+1 dimensional gravitational theory is dual to a d dimensional quantum field theory at its boundary.

[4] The calculation that led to Hawking radiation was in the semi-classical limit where $\hbar \to 0$. Hawking treated the radiation quantum-mechanically, but still treated the (curved) spacetime classically.

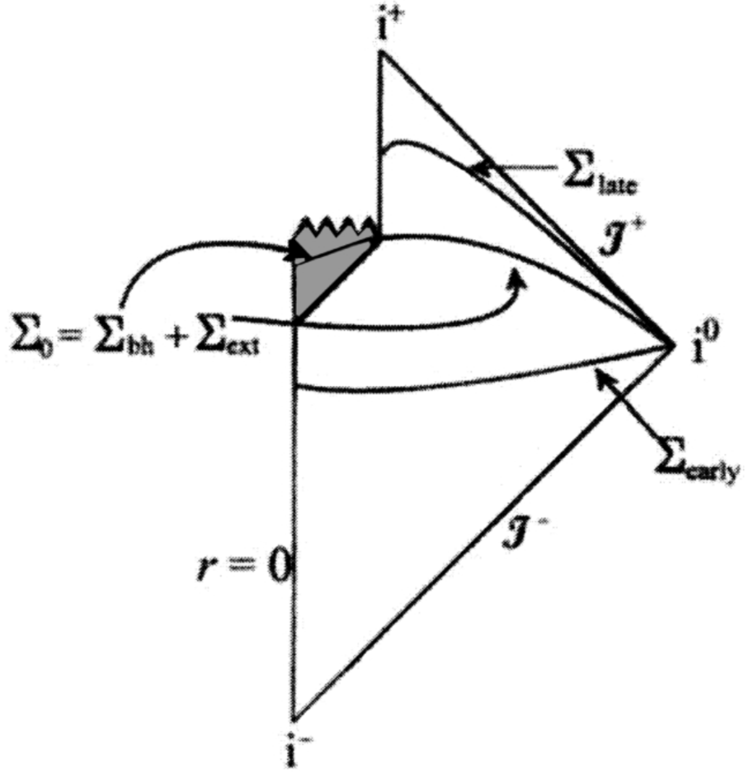

Figure 1: Due to quantum correlations between the field values localized inside and outside the black hole horizon, the exterior state on Σ_{ext} will be mixed. Due to which the late-time state on Σ_{late} will also have to be mixed, even if the global state on Σ_{early} was pure. In Quantum Mechanics we have unitarity where if you start with a pure state you will have a pure state after evolution, thus this process cannot be described by Qunatum Mechanics. [**5**]

What this means is irrespective of what goes into a black hole, the radiation that comes out will be a mixed state ρ. Hawking argued that this prediction, where black holes are emitting radiation and as a consequence shrinking and eventually evaporating, cannot be described by unitary evolution. See Figure [**fig:Figure 1**]. Now this is a big problem. We understand that under the laws of physics everything evolves with a unitary transformation U such that $U^\dagger \psi U = \psi$. Imagine you have a pure state ψ and you throw that into the black hole. Now what comes out is a mixed state ρ. Which is absurd since there does not exist any unitary transformation which turns a pure state into a mixed state. This made Hawking conclude that black holes are an exception to the laws of physics[5].

Reversibility is an important part of physics and the only exception to this rule is in the Copenhagen

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[5] As a reader this should make you laugh/furious because as physicists we know that the laws of physics are what holds everything together and if there are exceptions to them, it means they are not fundamental and we need to make new laws with *no* exceptions.

Interpretation of Quantum Mechanics when you make a measurement[6]. Eventually Hawking realised that there was a flaw in this approach and thought that a complete theory of quantum gravity maybe able to solve this since his calculations were a semi-classical approximation.

2.2 The Xeroxing Problem

In physics we have a crucial theorem which says that if we have an unknown quantum state, it is impossible to create another copy of the same system. This is called the **No Cloning Theorem**[7].

Consider two observers, Alice and Bob. Let Alice be outside the black hole and Bob inside near the singularity such that if Alice throws a qubit into the black hole, Bob will see that qubit come in. Now if one can somehow make unitarity possible and assume that all the information going inside the black hole will eventually come out, then we will see the qubit that Alice dropped in, come out through Hawking radiation. However for Bob, that qubit comes in but never leaves. This tells us that if all the information that goes in, somehow manages to come out, it will lead to having two copies of the same qubit, which violates the No cloning theorem. This is known as the *Xeroxing Problem.*

Suskind and others came along to solve this using *Black hole Complementarity.* According to Black hole complementarity , from Alice's perspective she views the black hole as a heated membrane situated right above the event horizon. This membrane has a finite number of degrees of freedom, given by the Bekenstein entropy of the black hole, and an area one Planck unit (10^{-66}cm^2) larger than the event horizon. If she drops anything into the black hole, the information will interact violently with this membrane and will eventually be re-emitted to the exterior universe, keeping Σ_{late}, pure. However from Bob's perspective while freely falling into the black hole, he would not notice anything unusual at the horizon, intead he would keep falling till he reaches the singularity without encountering any violent interaction with the horizon. So the question is, 'Who is right?'. Susskind and others say, *both.* Alice's observation is *complementary* to Bob's. They argued that the qubit that comes out as the Hawking radiation isn't really a cloned qubit

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[6] Other interpretations of Quantum Mechanics like the Many-Worlds interpretation where, if you make a measurement the world splits into different worlds based on the probabilities of the outcomes, for example, when you flip a coin, the probability of getting heads or tails is 1/2, now if you get heads it means in the other world you got tails, indicating that both possibilities do take place and thus emphasising on determinism. In the many world interpretation reversibility holds even during measurement[**4**].
There is a fun app you can try called *Universe Splitter* that splits the Universe according to the Many-Worlds interpretation.

[7] It is important to note here that, no cloning theorem doesn't mean that there cannot exist two or more copies of the same state. As long is we *know* what we are creating a copy of, we can create multiple copies.

since the same observer never sees both copies of the qubit. One might ask, what if Alice waits for the qubit to come out and then quickly goes inside to look at the other copy of the qubit? Well if you believe in math, by the time the qubit comes out as Hawking radiation, the qubit that went '*inside*' would have long hit the singularity and Alice would not be able to access it. Thus we can say that the qubit that comes out is the same that went inside, just viewed in two different ways.

2.3 The Firewall Paradox

Black hole complementarity heavily relies on the accurate experiences of both Alice and Bob. This brings about another puzzle in our understanding of the paradox. For this we would need physicists most reliable tool, Quantum Field theory (QFT). According to QFT the vacuum state of our Universe is characterized by an immense amount of short-range entanglement. What this tells us is that if someone were to observe a photon of Hawking radiation just emerging from the event horizon of a black hole, there must be an entangled photon just inside the event horizon[8]. If however the observer doesn't perceive these entangled pairs as they cross the event horizon, they would not see a smooth spacetime and instead the observer would encounter an abrupt "end of spacetime" marked by an extremely high-energy wall of photons, known as the **firewall** which would lead to the immediate disintegration of the observer. To comprehend this further, we would need to look at black holes from an information theoretic point of view. Black holes are considered to be the fastest scramblers. Consider n qubits in some state $|I\rangle$ that go into the black hole, when these qubits come out of the black hole, in the form of Hawking Radiation, we can model it by a *random* quantum circuit C applied to the qubits and think of the output as some pseudorandom pure state. Let the density matrix of n qubits be ρ,

$$\rho = \sum_{I=1}^{n} \lambda_I \left|I\right\rangle \left\langle I\right|$$

We look at the reduced density matrix of first k qubits,

$$\rho_k = \mathrm{Tr}_{n-k}\, \rho$$

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[8] One can think of this as the formation of huge numbers of Bell pairs at the Horizon.

Now we have two cases,

1. $k < n/2$. In this case ρ_k is very close to the maximally mixed state $|I\rangle_k$.
2. $k > n/2$. In this case ρ_k is no longer maximally mixed.

But what happens when exactly half of the qubits come out? At this stage Alice, our outside observer, will start to see entanglement, between the Hawking photons themselves, and between them and the infalling matter. The entropy of this entanglement is called *entanglement entropy.* The time at which exactly half of the qubits come out or when this entanglement entropy is maximum, is called the *Page time.* Now as the black hole slowly evaporates, suppose $2n/3$ of the qubits have come out in the form of Hawking radiation, and $n/3$ still remain. This gives us three subsystems,

1. A, $k = 2n/3$ qubits that have come out,
2. B, the very next qubit coming out,
3. C, $n/3$ qubits that are still remaining inside the black hole.

Now, we expect B to have some entanglement with A and further we know that the combined states $k+1$ is not a maximally mixed state. Consider transforming A unitarily. Now this will lead to Alice observing one qubit from A in an entangled pair with B. Thus being in a bell state. After confirming this entangled pair, if Alice immediately jumps inside the black hole, she will find another qubit inside C which is maximally entangled with B. However this violates *Principle of Monogamy of Entanglement.* The same qubit B cannot be maximally entangled with two other qubits. Thus all of this that Alice observes/experiences cannot be described by quantum mechanics.

Therefore if we want to preserve quantum mechanics, either Alice must be unable to observe entanglement between A and B or between B and C. This is called the **firewall paradox**.

How then do we solve this paradox? We can consider either of the following.[9]

1. Forget about black holes and concentrate only on the Universe apart from Black holes.

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[9] These ways are considered by S. Aronson in [**1**] and even though they make sense in some way, they aren't the only ones. The reader (and even the author) of this article are encouraged to find more solutions to the paradox.

2. Firewall really exists and acts as the end of spacetime and thus everything is fine for the spacetime.
3. Give up on Unitarity.
4. We take complementarity seriously and therefore C doesn't exist. C is just A and B measured in different basis.

For most physicists option 4 seems the most viable, however the thing about black holes is that the more we learn about them, the more we find how much more there is to learn.

3 Conclusions: A new idea of Islands

In the last decade a lot of progress has been made to understand and potentially solve the information paradox. Here I will talk about the recent attempts from experts. In thermodynamics and statistical mechanics there are two ways to define entropy; coarse-grained entropy, where we consider the system at a macroscopic level and fine-grained entropy where we consider the system at a microscopic level. Bekenstein-Hawking entropy is considered as the former since it increases under time evolution. There is also a gravitational formula for the von Neumann entropy which is given by gravitational path integral. Similar to the Feynaman path integral where we sum over all the possible paths that a particle can take, Gravitational path integral sums over all the possible geometries that a black hole can go through. Gravitational formula for the von Neumann entropy is similar to the Bekenstein-Hawking entropy, the only difference is the choice of the dividing surface. We choose a surface such that the total entropy of a black hole is minimized. This minimal value is the fine-grained entropy.

We now apply this fine-grained entropy to the evaporating black hole which is giving out hawking radiation. We use the gravitational path integral for an evaporating black hole, which sums over all the geometries that a black hole goes through while radiating Hawking-Radiation, and find the fine-grained entropy. Further, we know that this radiation is entangled with the fields in the interior of the black hole. Therefore if we want to minimize the '*total*' entropy, we would have to add the entropy of the inside region as well. This fine-grained entropy accurately follows the *Page curve*. *Page curve* is the curve that entropy of the system traces with time where the system reaches maximum entropy at *Page time*. Note that any approach

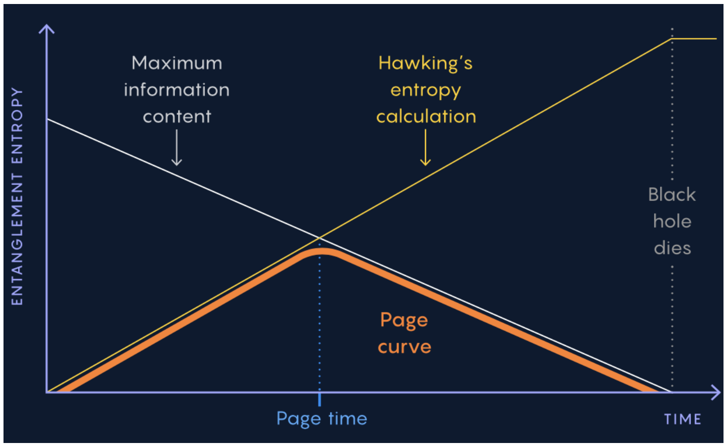

Figure 2: Amount of Hawking's entropy of a black hole and its entanglement entropy.[7]

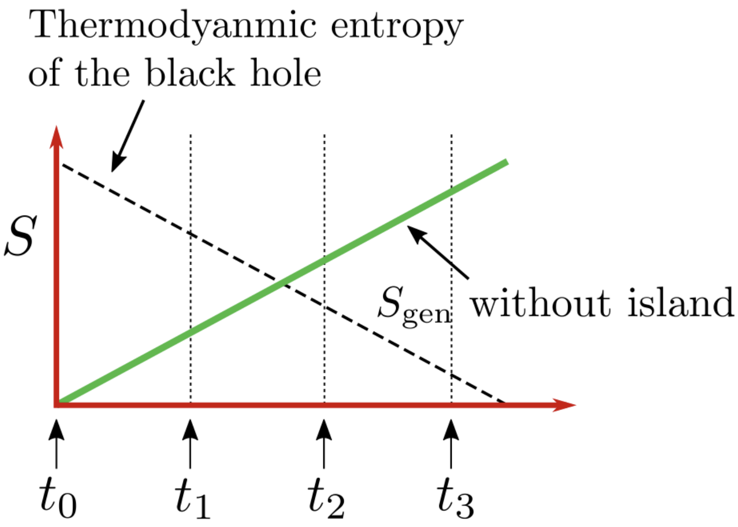

Figure 3: We consider here the Generalised entropy with and without islands [6]

that claims to solve the paradox must reproduce the page curve. See Figure 2. Therefore the generalised entropy of the radiation is the combined entropy of two regions. The region of radiation itself and the region inside of the black holes called *Islands*. According to this idea, information that falls into a black hole is not lost but instead gets encoded in subtle correlations between the interior and exterior of the black hole.

The idea of islands as a way to solve the paradox is very intriguing because what it's essentially saying is that an evaporating black hole goes through many geometries and one of them is where the black hole is connected, via wormholes, with replicas of the same black hole and the entropy of one black hole is calculated by calculating the entropy of n black holes and taking $n \to 1$[10]. Note that these wormholes don't really exist but just like Feynman Path Integrals, where we sum over all paths even those with negligible probability, the possibility of the wormhole geometry existing, alters the entropy of the black hole.

Note to the reader

Black holes are the most simple yet mysterious objects and a thorough understanding of them requires much more rigorous knowledge of complicated mathematical tools. The ideas presented here are in no way rigorous and are only provided for a more physical understanding.

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[10] This is also called Renyi Entropy

4 References

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[6] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, A. Tajdini, “The entropy of Hawking radiation”

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